membrane stiffness contribution to frequency becomes high compared to the bending stiffness contribution. Similar arguments apply to the transverse shear model as bending stiffness is introduced into the model.

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Reply by Authors to A. H. Flax

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BY developing an exact solution to Eq. (1) of Ref. 1 for the special case of uniform panel thickness, Dr. Flax points out that pure transverse shear panels (PTSP) cannot undergo flutter in the range of supersonic velocities for which piston theory applies. This is a paradoxical result, since it appears by considering, for example, the flutter of a sandwich-type panel consisting of two thin faces (modeled in accordance with Love-Kirchhoff theory), separated by a core modeled as a PTSP. As is well known, such a panel will flutter if the dynamic pressure becomes large enough. As the faces become thinner, the panel will flutter at lower dynamic pressures, i.e., become less stable. On the other hand, when in the limit the thickness of the faces vanishes (resulting in a structure with no bending stiffness), the flutter speed of the remaining PTSP becomes infinite according to Flax's exact solution. A paradox is thus reached in which a PTSP in high supersonic flow should be always stable, while more rigid structures (in bending) can flutter. This result belongs to the same class of aeroelastic paradoxes as the well-known membrane flutter paradox (see Refs. 2-4 of the Comment and our Refs. 2-4).

However, by using a singular perturbation method, a solution of the membrane flutter paradox has been developed in Ref. 5. This solution (which will be also useful for the problem at hand) considers the case of two-dimensional flat thin panels subjected to chordwise in-plane tensile stresses σ_x and exposed over its upper face to a supersonic flowfield. It is assumed that $D \equiv E' h^3$ is the panel bending stiffness, where $E' \equiv E/12(1-\nu^2)$ denotes the reduced Young's modulus. Linear piston aerodynamics is employed. An "interior solution" is first constructed by removing the exponential factor $\exp(\alpha x)$ appearing in the exact membrane flutter solution (a similar factor appears in the exact PTSP solution of the Comment); subsequently, the bending stresses near the edges are accounted for by a "boundary-layer" approach, and by letting $D \rightarrow 0$, the following flutter criterion is obtained

$$\frac{\rho_\infty U_\infty^2}{M_\infty \sigma_x}\left(\frac{E'}{\sigma_x}\right)^{1/2} = \left(\frac{2}{3}\right)^{3/2} \tag{1}$$

The above flutter criterion for "zero-thickness plates"[5] was obtained as a zero-order solution in the perturbation process. In Ref. 6 the same result was obtained from Erickson's three-dimensional panel flutter solution by a limiting process $D \rightarrow 0$. Note from Eq. (1) that the geometrical and mechanical characteristics in the spanwise direction are not intervening. Having in view the similarity between the aeroelastic equilibrium equations of flat panels with in-plane tensile stresses and sandwich-type panels with PTSP core, a flutter criterion for uniform PTSP can be obtained from Eq. (1) by replacing σ_x by G_{13}, thus becoming

$$(\Lambda_0)_* = \left(\frac{2}{3}\frac{G_{13}}{E'}\right)^{2/3} \tag{2}$$

where $\Lambda_0 = \kappa p_\infty M_\infty / E'$ defines a velocity parameter and E' denotes the reduced Young's modulus of the faces whose thickness becomes zero in the limiting process. Therefore, finite flutter speeds are predicted by viewing the uniform PTSP as the core of a symmetrical sandwich structure with the thickness of the faces tending to zero.

In the numerical example presented in Ref. 1, the critical flutter speed $(\Lambda_0)_*$ of the uniform PTSP—which intervenes as a fixed parameter in the optimal solution—was obtained by a Galerkin technique, where the representation of w corresponds to the interior solution of Ref. 5. It is of course advisable to use criterion (2)—believed by its authors[5] to be a crude but conservative one—for calculating the flutter speed of the uniform PTSP. It is hoped that the present discussion prompted by the Comment will stimulate the development of even more accurate solutions to the membrane and PTSP flutter problems by using higher order approximations in the perturbation method. This will also constitute better input data for the aeroelastic optimization problem considered in Ref. 1.

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The Nondimensional Coefficient of Thermal Conductivity

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THE purpose of this Note is to draw attention to the misleading notation of the coefficient of thermal conductivity divided by Prandtl number, κPr^{-1}, in many

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papers presented at recent AIAA meetings and in some published in the *AIAA Journal* (see Tables 1 and 2).

In the mid-1970s, Steger[1] developed a two-dimensional, time-dependent Navier-Stokes code; Schiff and Steger[2] developed a parabolized Navier-Stokes code; and Pulliam and Steger[3] developed a three-dimensional, time-dependent code. These three codes are based on a thin-layer approximation[4] of the viscous terms and use the Baldwin and Lomax[4] turbulence model with the Beam and Warming factored implicit scheme.[5] They have been in continuous use at NASA Ames and have been distributed to many research institutes. The codes have been modified, extended, and applied to a wide variety of problems. There have been at least 40 papers presented at meetings or published in journals that use basically the same technique. In these papers, except those that do not write it out explicitly (e.g., Ref. 6), the viscous term of the energy equation is typically written as [see Eq. (1), Ref. 1],

$$S_4 = u\tau_{xy} + v\tau_{yy} + \kappa Pr^{-1}(\gamma-1)^{-1}(a^2)_y \qquad (1)$$

Table 1 Papers presented at AIAA meetings

Paper	Author(s)
83-034	Degani and Schiff
83-224	Sahu, Nietubicz, and Steger
83-226	Hsieh
83-235	Chyu and Ono
83-237	Nietubicz, Sturek, and Heavey
83-462	Deiwert
82-035	Chyu and Kuwahara
82-101	Nietubicz
82-290	Rizk, Chaussee, and McRae
82-1341	Sturek, Guidos, and Nietubicz
82-1358	Sahu, Nietubicz, and Steger
81-050	Chaussee, Patterson, Kutler, Pulliam, and Steger
81-282	Davy, Green, and Lombard
81-1208	Srinivasan, Chyu, and Steger
81-1261	Rizk, Chaussee, and McRae
81-1262	Nietubicz
81-1900	Sturek and Mylin
80-063	Kutler, Pedelty, and Pulliam
80-066	Schiff and Sturek
80-067	Chaussee and Pulliam
80-1586	Sturek and Schiff
79-010	Nietubicz, Pulliam, and Steger
79-130	Schiff and Steger
79-134	Steger and Bailey
78-010	Pulliam and Steger
78-213	Kutler, Chakravarthy, and Lombard
77-665	Steger

Table 2 Papers published in the *AIAA Journal*

Paper	Author(s)
Vol. 20, Dec. 1982, p. 1724	Sturek and Schiff
Vol. 19, Feb. 1981, p. 153	Chaussee and Pulliam
Vol. 19, June, 1981, p. 684	Chyu, Davis, and Chang
Vol. 18, Feb. 1980, p. 159	Pulliam and Steger
Vol. 18, Dec. 1980, p. 1421	Schiff and Steger
Vol. 16, July 1978, p. 679	Steger

Here, κ is defined as the coefficient of thermal conductivity (normalized by the freestream value) and Pr is the Prandtl number. There is also a viscosity coefficient (normalized by the freestream value) μ used in these papers. This form of Eq. (1) first appeared in a paper by Peyret and Viviand.[7] It was adopted by Steger[1] and then apparently was just copied from one paper to another. Tables 1 and 2 contain lists of papers presented at AIAA meetings or published in the *AIAA Journal*. All of these papers use some form of Eq. (1). The list is by no means all-inclusive. We are concerned here only about the confusion caused by the heat conduction term containing the coefficient of thermal conductivity divided by the Prandtl number, κPr^{-1}.

While the dimensional coefficient of thermal conductivity κ^* is related to the dimensional coefficient of viscosity μ^* through the Prandtl number as $\kappa^* = \mu^* c_p / Pr$, the κ used in Eq. (1) is a dimensionless quantity and, for constant Pr, is equal to the dimensionless coefficient of viscosity μ—i.e., $\kappa = \mu$. For the turbulent flow computation, the laminar flow coefficients are replaced by

$$\mu \Rightarrow \mu_\ell + \mu_t \qquad (2)$$

$$\frac{\kappa}{Pr} = \frac{\mu}{Pr} \Rightarrow \frac{\mu_\ell}{Pr} + \frac{\mu_t}{Pr_t} \qquad (3)$$

Here, μ_t represents the turbulent eddy viscosity and Pr_t represents the turbulent Prandtl number. There should be no confusion when Eq. (2) is used in the viscous stress terms and Eq. (3) is used in the heat conduction terms. The use of the notation κ in these papers is redundant, confusing, and, in some cases (i.e., Refs. 8 and 9), erroneous. Most of the papers mentioned above either do not define κ (e.g., Ref. 1) or define κ as the coefficient of thermal conductivity (e.g., Refs. 2 and 3) and then merely refer to the Baldwin-Lomax turbulence model. References 8 and 9 state (p. 2, Ref. 8 or p. 4, Ref. 9).

"In the heat-flux terms the quantity $\kappa / c_p = \mu / Pr$ is computed as

$$\mu_{\text{laminar}} / Pr + \mu_{\text{turbulent}} / Pr_t \text{ where...''}$$

This error is caused by confusing the dimensionless quantities with dimensional ones. However, none of these papers explicitly expressed (correctly) how the dimensionless quantity κ is evaluated or mentioned that $\kappa = \mu$.

It is suggested that the readers of these papers replace the dimensionless thermal conductivity coefficient κ with the dimensionless viscosity coefficient μ in the energy equation. This will remove unnecessary repetition and will also make the nondimensional energy equation consistent with its dimensional form. In particular, it would avoid the appearance of dividing the thermal conductivity by a Prandtl number. Further, it is suggested that, if the notation given in Eq. (1) is used in future papers, the authors point out that the dimensionless coefficient of thermal conductivity is equal to the dimensionless coefficient of viscosity, i.e., $\kappa = \mu$, for constant Prandtl number.

It should be mentioned that we are solely concerned here with the form in which the heat conduction terms have been expressed within these papers. To the best of this author's knowledge, no error has been made associated with the dimensionless coefficient of thermal conductivity κ within the computer codes used to carry out the calculations.

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Errata: "A Theoretical and Experimental Investigation of a Transonic Projectile Flowfield"

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THE words "schlieren photo" should be replaced with "spark shadowgraph" in paragraph 1.

The figures have been incorrectly labeled. The figure captions should read:

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Fig. 1 Spark shadowgraph of projectile at $M = 0.98$.

Fig. 2 Physical grid for Navier-Stokes computations. a) Full grid, b) expanded grid.

Fig. 3 Triple deck model of shock/boundary-layer interaction.

Fig. 4 Boattail model configuration.

Fig. 5 Afterbody of wind tunnel model showing probe support mechanism.

Fig. 6 Comparison of Navier-Stokes, composite, and experimental surface pressure coefficients; $M = 0.94$.

Fig. 7 Velocity profiles at $X/D = 5.05$, 5.36, and 5.61 for $M = 0.94$.

Fig. 8 Comparison of displacement thickness: Navier-Stokes, composite, and experiment; $M = 0.94$.

Fig. 9 Comparison of Navier-Stokes, composite, and experimental surface pressure coefficients; $M = 0.97$.

Fig. 10 Velocity profiles at $X/D = 5.05$, 5.36, 5.49, and 5.61 for $M = 0.97$.

Fig. 11 Boattail shock formation from computed Mach number contours and schlieren photo.

Fig. 12 Comparison of skin friction coefficient between Navier-Stokes and composite solution, $M = 0.97$.